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Dust-lower-hybrid instability in a dusty plasma with a background of neutral atoms and streaming electrons and ions

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The dispersion relation and damping of an electrostatic dust-lower-hybrid mode has been derived using both the fluid and kinetic models of plasmas in magnetized dusty plasmas with a background of neutral atoms and streaming electrons and ions. This mode can be excited in a laboratory experiment when the streaming velocity of electrons and ions in the direction of the magnetic field exceeds the parallel phase velocity.
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Dusty plasmas consisting of electrons, ions, and micron- or submicron-sized dust grains usually occur in a background of neutral atoms in space and laboratory plasmas [1,2]. The dynamics of relatively massive ($m_d/m_i \sim 10^6 - 10^{12}$) and highly charged ($Z_d \sim 10^1 - 10^5$) dust grains can introduce new time and space scales leading to new waves, instabilities, and related phenomena. In unmagnetized dusty plasmas, a low-frequency (~ 15 Hz) and long-wavelength (~ 1 cm) mode known as a dust-acoustic (DA) wave has been extensively studied both theoretically [3–5] and experimentally [6–8] without considering the importance of the neutral background of such a plasma. Recently, instabilities of dust-acoustic waves in a plasma with a significant background pressure of neutral atoms have been investigated by Kaw and Singh [9]. Moreover, a magnetic field is invariably present in space plasma systems or can be applied for experimental purposes in laboratory plasmas. The streaming of electrons and/or ions relative to charged dust particles can induce dusty plasma modes in the presence of a magnetic field [10]. The situation can be easily achieved in a laboratory experiment by applying an external static magnetic field in the direction of ion flow in the sheath region where charged dust particles form a lattice.

In this Rapid Communication, we study the low-frequency dust-modes, particularly, the electrostatic dust-lower-hybrid (DLH) wave whose frequency can be higher than that of the DA wave and can be easily excited and measured in laboratory plasmas without much difficulty.

For the usual laboratory plasma temperature, the Larmor radii of electrons and ions will be small compared to the wavelengths of the waves under consideration and the massive dust grains can be taken as an unmagnetized fluid. Fluid equations governing the excitation of the plasma modes in general are

(i) *Momentum balance equation*:

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha = -\frac{q_\alpha}{m_\alpha}\nabla\phi + \mathbf{v}_\alpha \cdot \boldsymbol{\omega}_{c\alpha} - \frac{v_{t\alpha}^2}{n_{\alpha o}}\nabla n_\alpha - \nu_\alpha \mathbf{v}_\alpha ; \tag{1}$$

(ii) *Equation of continuity*:

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0; \tag{2}$$

(iii) *Poisson's equation*:

$$\nabla^2 \phi = -4\pi \sum_\alpha q_\alpha n_\alpha , \tag{3}$$

where $\omega_{c\alpha} = q_\alpha B_s / m_\alpha c$, $v_{t\alpha} = (T_\alpha/m_\alpha)^{1/2}$; q_α, m_α, T_α, $\nu_\alpha, n_{\alpha o}$ are the charge, mass, temperature, average collision frequency, and equilibrium number density of the species α, respectively, and B_s, ϕ, and c are the external static magnetic field, electrostatic potential, and the velocity of light.

For $n_{io}/n_n \ll 1, n_{do}/n_n \ll 1$ (n_n is the neutral density), we can take $\nu_{en} > \nu_{ei}$, ν_{ed} and $\nu_{in} > \nu_{dn}$, where $\nu_{\alpha n}$ is the average collision frequency of the particle α with the neutrals and the definitions of ν_{ei}, ν_{ed}, and ν_{in} are implied. Therefore, we may neglect electron or ion momentum loss due to $e-i$, $e-d$, and $i-d$ collisions.

Solving Eqs. (1)–(3) by the usual technique, we obtain the linear dielectric function as

$$\epsilon(\omega, \mathbf{k}) = 1 + \sum_{\alpha = e,i,d} \chi_\alpha , \tag{4}$$

where

$$\chi_\alpha = \left[\frac{k_\perp^2}{k^2} \frac{\omega_{p\alpha}^2}{\omega_{c\alpha}^2 - \omega'^2} \frac{\omega'}{\omega - k_\parallel u_{o\alpha}} - \frac{k_\parallel^2}{k^2} \frac{\omega_{p\alpha}^2}{\omega'(\omega - k_\parallel u_{o\alpha})} \right] \times \left[1 + \frac{k^2 v_{t\alpha}^2}{\omega_{p\alpha}^2} \left(\frac{k_\perp^2}{k^2} \frac{\omega_{p\alpha}^2}{\omega_{c\alpha}^2 - \omega'^2} \frac{\omega'}{\omega - k_\parallel u_{o\alpha}} - \frac{k_\parallel^2}{k^2} \frac{\omega_{p\alpha}^2}{\omega'(\omega - k_\parallel u_{o\alpha})} \right) \right]^{-1} . \tag{5}$$

Here, $\omega_{p\alpha}^2 = (4\pi q_t^2 n_{\alpha o}/m_\alpha)^{1/2}$, $\omega' = \omega - k_\parallel u_{o\alpha} + i\nu_{\alpha n}$, and $u_{o\alpha}$ is the constant drift velocity of the species α. The symbol $\parallel(\perp)$ denotes a parallel (perpendicular) component relative to the direction of the magnetic field ($\mathbf{B}_s \parallel \hat{\mathbf{z}}$).

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For the low-frequency electrostatic dust-lower-hybrid mode propagating nearly perpendicular to the magnetic field with $\omega_{cd} \ll \omega \ll \omega_{ci} \ll \omega_{ce}$ and $kv_{t\alpha} \ll \omega_{p\alpha}$, we obtain from Eq. (5),

$$\chi_e \simeq \frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1+\frac{i\nu_{en}}{\omega-k_\parallel u_o}\right) - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{(\omega-k_\parallel u_o)(\omega-k_\parallel u_o+i\nu_{en})},$$

$$\chi_i \simeq \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{i\nu_{in}}{\omega-k_\parallel u_o}\right) - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{(\omega-k_\parallel u_o)(\omega-k_\parallel u_o+i\nu_{in})}, \tag{6}$$

$$\chi_d \simeq -\frac{k_\perp^2}{k^2}\frac{\omega_{pd}^2}{\omega'^2}\left(1+\frac{i\nu_{dn}}{\omega}\right) - \frac{k_\parallel^2}{k^2}\frac{\omega_{pd}^2}{(\omega-k_\parallel u_o)(\omega-k_\parallel u_o+i\nu_{dn})},$$

where the electrons and ions are assumed to have the same drift velocity, u_o.

For $u_o=0$ and $\nu_\alpha=0$, the linear dispersion relation of the DLH mode is obtained for nearly perpendicular propagation, $k_\perp \gg k_\parallel$ as

$$\omega^2=\omega_{DLH}^2\left[1+\frac{k_\parallel^2}{k^2}\frac{n_{eo}m_d}{Z_d^2 n_{do}m_e}\left(1+\frac{n_{io}m_e}{n_{eo}m_i}\right)\right], \tag{7}$$

where

$$\omega_{DLH}^2=\omega_{cd}\omega_{ci}\left(\frac{Z_d n_{do}}{n_{io}}\right)\left(1+\frac{n_{eo}m_e}{n_{io}m_i}\right)^{-1}. \tag{8}$$

For the collision dominated plasmas, we assume ν_{en}, $\nu_{in} > (\omega - k_\parallel u_o)$, and $\omega^2 \gg \nu_{dn}^2$ for the cold dust. Using Eqs. (6), the dispersion relation of the DLH mode is given by

$$\omega^2=\omega_{DLH}^2\left[1+\frac{k_\parallel^2}{k^2}\frac{\omega_{ci}^2}{\nu_{in}^2}\left(1+\frac{n_{eo}T_i}{n_{io}T_e}\right)\right], \tag{9}$$

where we used $\nu_{in}/\nu_{en}=(T_i m_e/T_e m_i)^{1/2}$.

Using Eqs. (6), the damping rate of this mode for $k_\perp \gg k_\parallel$ is obtained as

$$\frac{\gamma}{\omega}=-\frac{1}{2}\left[\frac{\nu_{dn}}{\omega}+\frac{\nu_{in}}{\omega-k_\parallel u_o}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left\{1+\frac{n_{eo}}{n_{io}}\left(\frac{m_e}{m_i}\frac{T_e}{T_i}\right)^{1/2}\right\}\right]. \tag{10}$$

Thus, the DLH mode can grow when $u_o > \omega/k_\parallel$ with the growth rate determined by Eq. (10). It is noticed from the above equations that the dynamics of electrons is not important for the ion-dust hybrid wave.

To study the kinetic instability of the dust-lower-hybrid wave under consideration in a dusty plasma in the collisionless limit with electrons and ions drifts relative to the dust along the direction of the external magnetic field, we obtain the real and imaginary parts of the dielectric function using the Vlasov-kinetic equation [11]

$$\epsilon_r \simeq 1+\frac{1-\Gamma_{oi}}{k^2\lambda_{Di}^2}+\frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{\omega_{pe}^2}{\omega_{pi}^2}\frac{\omega_{ci}^2}{\omega_{ce}^2}\right) - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2+\omega_{pi}^2}{(\omega-k_\parallel u_o)^2}-\frac{\omega_{pd}^2}{\omega^2}, \tag{11}$$

$$\epsilon_i=\sqrt{\frac{\pi}{2}}\left\{\frac{\omega-k_\parallel u_o}{k_\parallel v_{te}}\frac{1}{k^2\lambda_{De}^2}\exp\left[-\left(\frac{\omega-k_\parallel u_o}{\sqrt{2}k_\parallel v_{te}}\right)^2\right] + \frac{\omega-k_\parallel u_o}{k_\parallel v_{ti}}\frac{1}{k^2\lambda_{Di}^2}\exp\left[-\left(\frac{\omega-k_\parallel u_o}{\sqrt{2}k_\parallel v_{ti}}\right)^2\right]\right\}, \tag{12}$$

where $\Gamma_{oi}=I_0(b_i)\exp(-b_i)$, $b_i=k_\perp^2 v_{ti}^2/\omega_{ci}^2$, and I_0 is the zero-order modified Bessel function of the argument (b_i). We have considered electrons and ions magnetized and dust particles as cold and unmagnetized. Thus, the dispersion relation of the DLH wave for high density plasma is given by Eq. (7) and the damping rate of the mode is given by

$$\gamma_L=-\omega_{DLH}^2\epsilon_i/2\omega_{pd}^2, \tag{13}$$

where ω_{DLH} is defined by Eq. (8) and ϵ_i, by Eq. (12). Hence, we note that the instability occurs when $u_o > \omega/k_\parallel$ with the growth rate γ_L.

For the usual parameters in laboratory experiments, $m_d/m_i \sim 10^{12}$, $B_s \sim 1$ kG, one can obtain $\omega_{ci} \sim 10^6$ Hz and $\omega_{cd} \sim 10^{-2}$ Hz. Thus, the dust cyclotron frequency will be too small to be detected in the laboratory conditions. It may be significant in space environments. However, the dust-lower-hybrid frequency may take a significant value, $\omega_{DLH} \sim 10^2$ Hz for $Z_d n_{do}/n_{io}=1$. This frequency, which is at least one order higher than the frequency of the dust-acoustic mode, can be easily detected and may have significant effects in laboratory plasma experiments using a magnetic field.

Therefore, we propose a dusty plasma experiment using a magnetic field, where the dust-lower-hybrid wave can be easily excited and various features of dusty plasmas can be studied. We emphasize that the present work should be useful to understand the various aspects of dust-lower-hybrid waves and dust-Coulomb crystal formation in a laboratory experiment in the presence of a magnetic field as well as in regions where structures, such as stars, may form in interstellar and other astrophysical dusty environments.

To summerize, we have presented the kinetic and hydrodynamic instabilities of low-frequency electrostatic dust-lower-hybrid waves propagating nearly perpendicular to the applied magnetic field in a collisional and finite temperature dusty plasma. When the streaming velocity of electrons and ions in the direction of the magnetic field exceeds the parallel phase velocity, the dust-lower-hybrid mode can be excited in a laboratory experiment. The existence of this electrostatic dust-lower-hybrid mode can give rise to a host of phenomena in a magnetized dusty plasma. For example, the electrostatic noise at the dust-lower-hybrid frequency can be explained in terms of this mode.

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